Coastal Engineering Technical Note

ESTIMATING POTENTIAL LONGSHORE SAND TRANSPORT RATES USING WIS DATA

PURPOSE: To present a procedure to calculate potential longshore sand transport rates using Wave Information Study (WIS) Phase III hindcast wave estimates. Refraction and shoaling of incident linear waves are calculated using Snell's law and conservation of wave energy flux. A shallow-water wave breaking criterion defines wave properties at the break point, and potential longshore sand transport rates are calculated by means of the energy flux method in the Shore Protection Manual (SPM 1984).

INTRODUCTION: The longshore sand transport rate, Q, is the volumetric rate of sand movement parallel to the shore. Longshore transport is confined mainly to the surf zone and on an open coast is produced predominantly by waves breaking at an angle to the shoreline. It has been empirically determined that the longshore transport rate is proportional to a quantity $P_{\ell s}$, referred to as the longshore wave energy flux factor. The expression for $P_{\ell s}$ given by Equation 4-39 of the SPM (1984) is

$$P_{\ell s} = \frac{\rho g}{16} H_b^2 C_{gb} \sin(2\alpha_b) \tag{1}$$

where

ρ = density of water
g = acceleration due to gravity
H_b = significant wave height at breaking
C_{gb} = wave group speed at breaking
α_b = breaking wave angle

In shallow water,

$$C_{gb} \simeq \sqrt{g d_b} \tag{2}$$

where d_b is the depth at breaking. The expression for Q given by Equation 4-49 of the SPM (1984) is

$$Q = \frac{K}{(\rho_s - \rho) g a'} P_{\ell s} \tag{3}$$

where

K = nondimensional empirical sand transport coefficient ($K = 0.77$)
ρ_s = density of sediment (quartz sand, $\rho = 2.65$ g/cm^3)
a' = volume solids/total volume (accounts for sand porosity, $a' = 0.6$)

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Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **JUN 1989**	2. REPORT TYPE	3. DATES COVERED **00-00-1989 to 00-00-1989**

4. TITLE AND SUBTITLE **Estimating Potential Longshore Sand Transport Rates Using WIS Data**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **U. S. Army Engineer Wattways Experiment Station,Coastal Engineering Research Center,P.O. Box 631,Vicksburg,MS,39180**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**6**	

Equation 3 provides an estimate of the longshore sand transport rate in terms of wave quantities at breaking. The actual physical situation may preclude such transport, for example, along beaches where the sand supply is deficient. The quantity Q should therefore be viewed as the potential transport rate.

Substituting Eqs. 1 and 2 into Eq. 3 yields

$$Q = \frac{K}{16[(\rho_s/\rho)-1]a'} \sqrt{\frac{g}{\gamma}} \frac{H_b^{5/2}}{2.386} \sin(2\alpha_b) \tag{4}$$

for which it was assumed that breaking wave height was linearly related to the depth at breaking as

$$H_b = \gamma d_b \tag{5}$$

in which $\gamma = 0.78$ is the wave breaking index. In Eq. 4, the wave energy frequency and directional spectrum is represented as a single statistical wave that undergoes transformation as described by linear wave theory. The factor 2.386 converts the input significant wave height to root mean square wave height for compatibility with the $K = 0.77$ design value.

To calculate the potential longshore sand transport rate using Eq. 4, the breaking wave height and incident angle with respect to the shoreline are required. WIS hindcast estimates, however, are given for water depths greater than or equal to 10 meters (Jensen 1983). Therefore, a transformation of the WIS hindcast wave estimates to breaking conditions is necessary. Refraction and shoaling of incident waves provided by WIS is accomplished using linear wave theory. Primary assumptions are that offshore depth contours are straight and parallel to the trend of the shoreline and that no wave energy dissipation occurs prior to breaking. The governing equations are given below. The subscripts 1 and 2 denote values at the water depths d_1 and d_2.

Wave direction is obtained through Snell's law:

$$\frac{\sin(\alpha_1)}{L_1} = \frac{\sin(\alpha_2)}{L_2} \tag{6}$$

where L = wavelength. The wave height is obtained by invoking the conservation of wave energy flux directed onshore:

$$E_1 C_{g1} \cos(\alpha_1) = E_2 C_{g2} \cos(\alpha_2) \tag{7}$$

where E is the wave energy. Equations for the wave energy and group speed at an arbitrary depth are:

$$E = \frac{1}{8} \rho g H^2 \tag{8}$$

$$C_g = \frac{L}{2T} \left(1 + \frac{4\pi d}{L \sinh(4\pi d/L)}\right) \tag{9}$$

where T = wave period.

WAVE TRANSFORMATION PROCEDURE: The first step in the wave transformation procedure is to calculate the wavelength at the location where the wave height, incident angle, period, and water depth are known (denoted as location 1 in Eqs. 6 and 7). In linear wave theory, wavelength is given by

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right) \tag{10}$$

This equation can be solved by using a Newton-Raphson iteration. The deep-water expression $L_o = gT^2/2\pi$ is used as the initial trial value.

The second step is to determine wave height, water depth, and incident angle at breaking (denoted as location 2 in Eqs. 6 and 7). This requires use of the breaking wave criterion given in Eq. 5. Solving Eq. 6 for cos α_2, and substituting it and Eq. 5 into the right-hand side of Eq. 7 yields an expression for the conservation of wave energy flux in terms of the known wave characteristics (left-hand side of Eq. 7) and the unknown wave characteristics at breaking (right-hand side of Eq. 7). Equation 10 evaluated at breaking gives another expression in terms of the wavelength and water depth at breaking. A Newton-Raphson type solution can be used to iterate for the two unknown variables of wavelength and water depth at breaking. The initial trial value for the wavelength at breaking is L/4; the depth at breaking is initially set equal to the input wave height.

Having determined the wavelength and depth at breaking, breaking wave height is calculated using Eq. 5, and the breaking wave angle is calculated using Eq. 6. The potential longshore sand transport rate is then estimated using Eq. 4. (The described calculations can be performed with the FORTRAN program WISTRT.)

INPUT WAVE CONDITIONS: WIS hindcast wave estimates have been compiled at three types of stations: Phase I (deep water), Phase II (intermediate water), and Phase III (10-m depth). A Phase III station will be used in the examples provided. The concepts, however, are equally applicable to wave estimates obtained from WIS Phase I and Phase II stations.

WIS Report No. 9 (Jensen 1983) lists 20-year percent occurrence tables of wave statistics for 166 Atlantic coast stations. Additional reports are in preparation for the Gulf, Pacific, and Great Lakes coasts. The hindcast wave estimates are presented in 30-deg angle bands. Angles reported for WIS Phase III stations, θ_{WIS} , are defined with respect to shoreline orientation and are measured counter-clockwise from the shoreline (i.e., $0^{o} \leq \theta_{WIS} \leq 180^{o}$). For calculation of longshore sand transport, a right-handed coordinate system is more convenient, in which waves approaching normal to the shoreline are given an angle of 0 deg. Looking seaward, waves approaching from the right are associated with negative angles, and waves approaching from the left are

associated with positive angles such that positive transport is directed to the right. Conversion of WIS angles to angles associated with transport calculations, θ, may be accomplished by means of the following relationship:

$$\theta = \theta_{WIS} - 90^{0} \tag{11}$$

An example of a typical WIS Phase III wave statistics summary is given in Table 1. Percent occurrence multiplied by 1000 is listed for specific wave height and period bands. The header gives the record length (20 years), angle band, water depth, and shoreline orientation angle. Because of the sensitivity of the calculated longshore sand transport rate to shoreline orientation, this quantity should be verified using a nautical chart. The last line in the table gives the average and largest significant wave height, together with the angle class percent. A representative wave period for the given average significant wave height may be determined by calculating a weighted average of all the wave periods given across the bottom of the table in the row labeled "total." Similarly, a representative period for each of the wave height bands can be calculated. The central angle of the angle band given in Table 1 (75 deg), converted to the transport coordinate system, is -15 deg.

Data given in WIS statistical tables may be used in several ways to calculate the potential longshore sand transport rate. Two examples using the data in Table 1 will be given. In the first example, the potential longshore sand transport rate is estimated with average significant wave height. In Example 2, wave data are more accurately represented by calculating a representative wave period for each of the given wave height bands.

Table 1

Percent Occurrence by Angle Band, WIS Atlantic Coast Phase III Station 54

STATION 54 20 YEARS WAVE APPROACH ANGLE (DEGREES) = 60.0 - 89.9
SHORELINE ANGLE = 4.0 DEGREES AZIMUTH
WATER DEPTH = 10.00 METERS
PERCENT OCCURRENCE (X1000) OF HEIGHT AND PERIOD BY DIRECTION

HEIGHT (METERS)	PERIOD (SECONDS)										TOTAL
	0.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9	8.0-8.9	9.0-9.9	10.0-10.9	11.0-LONGER	
0.00 - 0.49	634	744	.	.	593	2861	2286	992	278	455	8843
0.50 - 0.99	.	571	1309	148	37	718	631	277	311	106	4108
1.00 - 1.49	.	.	111	693	147	164	131	30	70	58	1404
1.50 - 1.99	.	.	.	34	256	159	71	6	11	25	562
2.00 - 2.49	.	.	.	.	29	285	80	5	1	25	425
2.50 - 2.99	.	.	.	.	.	51	92	5	1	1	150
3.00 - 3.49	.	.	.	.	.	.	6	10	5	.	21
3.50 - 3.99	.	.	.	.	.	.	.	8	.	.	8
4.00 - 4.49	.	.	.	.	.	.	.	.	.	.	0
4.50 - 4.99	.	.	.	.	.	.	.	.	.	.	0
5.00 - GREATER	.	.	.	.	.	.	.	.	.	.	0
TOTAL	634	1315	1420	875	1062	4238	3297	1333	677	670	

AVERAGE HS(M) = 0.61 LARGEST HS(M) = 3.70 ANGLE CLASS % = 15.5

******************************** EXAMPLES ********************************

Example 1: Calculate the potential longshore sand transport rate using average significant wave height and a weighted average period for the data given in Table 1.

Program Input:

Given: $(H_s)_{avg}$ = 0.61 m
Depth = 10.0 m
Percent Occur. = 15.5

Calculated: $(T)_{avg}$ = 7.2 sec
θ_{WIS} = 75 deg
θ = -15 deg

Program Output: H_b = 0.87 m
θ_b = -5.6 deg
Q = -50,800 m^3/year (directed to the left)

Example 2: Calculate the potential longshore transport rate using each of the eight wave height bands given in Table 1.

	Program Input					Program Output		
Wave Condition	$(H_s)_{avg}$ m	$(T)_{avg}$ sec	θ deg	Depth m	Percent Occurrence %	H_b m	θ_b deg	Q m^3/year
1	0.25	7.5	-15.0	10.0	8.843	0.43	-3.9	-3,500
2	0.75	6.5	-15.0	10.0	4.108	1.00	-6.2	-21,100
3	1.25	6.6	-15.0	10.0	1.404	1.53	-7.6	-25,500
4	1.75	7.3	-15.0	10.0	0.562	2.09	-8.5	-24,700
5	2.25	7.9	-15.0	10.0	0.425	2.62	-9.3	-36,000
6	2.75	8.2	-15.0	10.0	0.150	3.12	-10.1	-21,200
7	3.25	9.5	-15.0	10.0	0.021	3.66	-10.7	-4,700
8	3.75	9.5	-15.0	10.0	0.008	4.13	-11.3	-2,500
								-139,200

Q = -139,200 m^3/year
(directed to the left)

DISCUSSION: The potential longshore sand transport rate calculated in Example 2 differs significantly from that calculated in the first example and is expected to provide a more accurate estimate because of the finer discretization of the available wave estimates. Examples illustrate calculations for one angle band only; similar calculations using WIS data from all angle bands would be used to determine the potential longshore sand transport rate for a particular site. It should be noted that actual transport rates are influenced by coastal structures and the availability of sand and will likely be lower than those estimated using the procedures presented above. Additional examples are given in Gravens (1988).

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ADDITIONAL INFORMATION: The program WISTRT is available on a diskette or as a printed listing and may be obtained from Mr. Mark Gravens (601) 634-3809, Mark.B.Gravens@erdc.usace.army.mil

REFERENCES:

Gravens, M. B. 1988. "Use of Hindcast Wave Data for Estimation of Longshore Sediment Transport," Proc. Svmposium on Coastal Water Resources, American Water Resources Association, Wilmington, NC, pp 63-72.

Jensen, R. E. 1983. "Atlantic Coast Hindcast, Shallow-Water Significant Wave Information," Wave Information Study Report 9, US Army Engineer Waterways Experiment Station, Vicksburg, MS.

Shore Protection Manual. 1984. 4th ed., 2 vols, US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, US Government Printing Office, Washington DC.